B.Sc Part II (Subsidiary).

Tangents and Normals:

Tangent:— Let P and Q be any two points on a given curve. When the point Q, while moving along the curve, ~~to~~ coincides with the point P, then the limiting position of the secant PQ is called the tangent at the point P. That is, a tangent is a straight line passing through two coincident pts.

Let the equation of the curve be given by $f(x, y) = 0$

We know ~~that~~ from the chapter on partial differentiation that

$$\frac{dy}{dx} = \frac{\partial f/\partial x}{\partial f/\partial y}$$

Therefore putting the value of $\frac{dy}{dx}$ on the equation $Y - y = \frac{dy}{dx}(X - x)$

We get

$$Y - y = - \frac{\partial f/\partial x}{\frac{\partial f}{\partial f}} (X - x)$$

or, $$(Y - y)\frac{\partial f}{\partial y} = -\frac{\partial f}{\partial x}(X - x)$$

$$(X - x)\frac{\partial f}{\partial x} + (Y - y)\frac{\partial f}{\partial y} = 0.$$

This is the equation of the tangent.

Normals:— The normal at any point (x, y) is that st. line which is perpendicular to the tangent at (x, y).

Thus to find out the equation of the normal at the point (x,y), we shall have to find out the equation of that straight line which passes through (x,y) and is perpendicular to the tangent at (x,y). We therefore assume that the equation of the normal is

$$Y-y=m(X-x) \quad \text{———} (1)$$

We know from the last article that the equation of the tangent at the point (x,y) is

$$Y-y=\frac{dy}{dx}(X-x) \quad \text{———} (2)$$

Now since (1) and (2) are perpendicular to each other, therefore

$$m\times\left(\frac{dy}{dx}\right)=-1 \quad \text{i.e. } m=-\frac{1}{\frac{dy}{dx}}.$$

putting $m=-\frac{1}{\frac{dy}{dx}}$ in (1), we get

$$Y-y=-\frac{1}{\frac{dy}{dx}}(X-x)$$

$$\Rightarrow \frac{dy}{dx}(Y-y)=-(X-x)$$

$$\Rightarrow (X-x)+(Y-y)\frac{dy}{dx}=0$$

This is the equation of the normal at the point (x,y).